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Monte Carlo series analysis of irreversible self-avoiding walks: II. The growing self-avoiding walk

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Abstract. We study the scaling behaviour of the recently introduced growing SAW. We apply a ratio type analysis of Monte Carlo generated enumeration data to extract the critical indices. For the two-dimensional walk we find the usual SAW values for these indices. For the three-dimensional walk the true asymptotic behaviour can only be studied for extremely long chains. However, on the basis of our data, we can exclude the possibility of an upper critical dimension $d_c = 3$.

1. Introduction

The study of kinetic growth phenomena has recently attracted considerable attention (Family and Landau 1984). In the first paper of this series (Kremer and Lyklema 1985b) we presented a detailed analysis of the indefinitely growing SAW (IGSAW, Kremer and Lyklema 1985a). This is a truly kinetic walk, which is constructed in such a way that it recognises cages, independent of the size of the cage. Another interesting model in this context is the subject of the present paper, the recently introduced growing version of the self-avoiding walk (GSAW, Majid *et al* 1984a, b, p 36, Lyklema and Kremer 1984a, b, Hemmer and Hemmer 1984). It has been suggested by these authors that this walk can be used as a model to study the growth process of a linear polymer in the case where the growth rate is much faster than the relaxation time of the polymer. From their study of this walk (called KGW instead of GSAW) Majid *et al* (1984a, b) conclude that it belongs to a different universality class from the usual self-avoiding walk (SAW). They conclude that ν, the exponent of the mean square end-to-end distance

$$\langle R^2(N)\rangle \sim N^{2\nu}$$

equals 0.66 in two dimensions and 0.50 in three dimensions. In addition they find an upper critical dimension (d_c) of three. Their results are obtained from Monte Carlo simulations, exact enumerations ($d = 2$ only) and a Flory type theory. The present authors have studied the same model (from now on called GSAW) on the square lattice and on the diamond lattice by exactly enumerating all possible walks up to $N = 22$ steps. We find $\nu = 0.68$ (Lyklema and Kremer 1984a, b) on the square lattice, in agreement with Majid *et al* (1984a, b). However their result $\nu = \frac{1}{2}$ and $d_c = 3$ is not confirmed by our exact enumeration on the diamond lattice. In a response to these results Peliti (1984) has given a field theoretic argument from which it follows that the GSAW is in the same universality class as the usual SAW. By considering the trapping

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probability Pietronero (1984) has argued that only walks which are much longer than the mean survival length can show the asymptotic behaviour. From this he concludes that the GSAW is in the SAW universality class and that the asymptotic behaviour can only be studied for very large N ($N \sim 10^6$ in three dimensions). To settle this dispute we have performed high precision Monte Carlo calculations for large enough N (2D: $N = 200$; 3D: $N = 700$) to see the asymptotic behaviour set in.

In the next section we give a definition of the GSAW and discuss the Monte Carlo simulation. In § 3 we present the numerical results and their analysis and in § 4 we give the conclusions and a summary.

2. The model

The growing self-avoiding walk is a combination of the usual SAW and the true SAW (Amit *et al* 1983) in the $g \to \infty$ limit. In this case the one-step probability is equal to the inverse of the number of nearest-neighbour sites with the lowest occupation number. If one combines this with the self-avoiding restriction that the walk can visit a site only once the result is the following definition for the one-step probability P_i

$$P_i = 1/\text{number of unoccupied sites.} \tag{1}$$

For the usual SAW, which describes the equilibrium properties of a polymer in a good solvent, each configuration has the same probability. Thus the one-step probability in this case is $P_i = 1/q_0$, independent of the surrounding. Here q_0 is the coordination number minus one. For instance, the probability to have a particular N step walk on the square lattice is $\frac{1}{4}(\frac{1}{3})^{N-1}$. If the walk violates the self-avoiding condition it is terminated and one has to start from the beginning. For the GSAW the probability to have a N step walk equals $\prod_{i=1}^{N} P_i$, with P_i defined in equation (1). Here again one has to stop the walk when the self-avoiding condition is violated, in contrast with the true SAW which cannot get trapped. To visualise the random process we have given in figure 1 the different transition probabilities which can occur on the square lattice. One clearly sees the irreversibility of the process. The probability to have the 12 step process leading from A to B is $\frac{1}{4} \times (\frac{1}{3})^9 \times \frac{1}{2} \times 1$ whereas the inverse trajectory has the probability of $\frac{1}{4} \times (\frac{1}{2})^5 \times (\frac{1}{3})^6$. The probability ratio of the two processes is $\frac{16}{27}$, that is the second process is almost twice as likely as the first one. The probability for this walk with the usual SAW rules is $\frac{1}{4}(\frac{1}{3})^{11}$. The probability ratio (A → B)/SAW is 4.5 and the ratio (B → A)/SAW is $(\frac{3}{2})^5 = 7.6$. So clearly the GSAW probabilities are greater that the SAW probabilities or, in other words, early termination is more likely to happen for the SAW than for the GSAW. However, it should be noted that in both cases exactly the

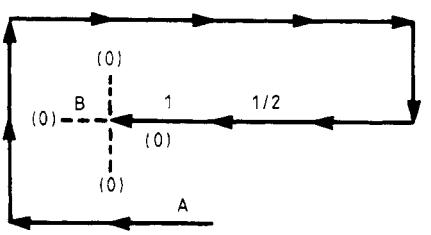

Figure 1. Example of a GSAW which is terminated. When the one-step probabilities differ from the SAW value $1/q_0$, it is indicated.

same trajectories occur. The case of the honeycomb lattice is special in the sense that the probability of a N step walk is the same in both directions. Here the one-step probability is 1 or $\frac{1}{2}$ depending on whether there is a nearest-neighbour contact or not. Because the number of nearest neighbours does not change by inverting the direction, the product probability does not change. Also one should realise that this model is not truly kinetic because of the finite termination probability.

For this model we have performed Monte Carlo simulations on the square lattice and on the diamond lattice. The diamond lattice is studied because it is the lattice with the smallest coordination number in three dimensions and one therefore expects to see the asymptotic behaviour earlier than on other three-dimensional lattices. We have used the simple sampling method (see Kremer *et al* 1982). This is a straightforward procedure in which one chooses the new site from all the previously unoccupied sites with equal probabiltiy. If no unoccupied site is available the chain is terminated and one repeats the procedure beginning at the origin. In this way we have generated a very large number of chains. In two dimensions we have generated chains up to a length of $N = 200$. Two typical values are: 48.5×10^6 for $N = 100$, acceptance rate 22.2% and 10.5×10^6 for $N = 200$, acceptance rate 2.6%. For a complete list see table 1. In three dimensions we have generated chains with a maximum length of $N = 700$. Three typical values are: 94.7×10^6 for $N = 100$, acceptance rate 96.7%; 34.5×10^6 for $N = 200$, acceptance rate 90.9% and 17.1×10^6 for $N = 700$, acceptance rate 62.5%. In table 2 we give the complete results for the three-dimensional GSAW. From these configurations we have calculated the mean square end-to-end distance

$$\langle (R^2(N)) \rangle = \langle (r_N - r_0)^2 \rangle \tag{2}$$

and the fourth moment

$$\langle R^4(N) \rangle = \langle R^2(N)^2 \rangle. \tag{3}$$

Table 1. Mean square displacement $\langle R^2(N) \rangle$ of the GSAW on the square lattice. Only the value for every fifth step is given.

N	$\langle R^2(N) \rangle$	N	$\langle R^2(N) \rangle$
5	0.000	10	21.844
15	36.391	20	52.420
25	69.622	30	87.994
35	107.299	40	127.703
45	148.882	50	171.002
55	193.878	60	217.603
65	242.029	70	267.236
75	293.078	80	319.659
85	346.875	90	374.800
95	403.323	100	432.530
105	462.270	110	492.672
115	523.642	120	555.207
125	587.285	130	619.951
135	653.157	140	686.891
145	721.047	150	755.826
155	791.089	160	826.948
165	863.164	170	899.967
175	937.245	180	975.061
185	1013.159	190	1051.898
195	1091.002	200	1130.665

Table 2. Mean square displacement $\langle R^2(N)\rangle$ of the GSAW on the diamond lattice. Only the value for every tenth step is given.

N	$\langle R^2(N)\rangle$	N	$\langle R^2(N)\rangle$
10	60.032	20	124.878
30	191.793	40	257.117
50	328.236	60	400.136
70	464.741	80	531.700
90	602.796	100	683.224
110	762.023	120	837.615
130	903.884	140	977.015
150	1051.791	160	1131.903
170	1214.152	180	1263.176
190	1336.428	200	1409.805
210	1483.345	220	1556.961
230	1630.692	240	1704.749
250	1778.772	260	1852.874
270	1927.134	280	2001.618
290	2076.091	300	2150.668
310	2225.431	320	2300.158
330	2375.163	340	2450.157
350	2525.128	360	2600.298
370	2675.733	380	2751.079
390	2826.485	400	2902.190
410	2977.728	420	3053.458
430	3129.377	440	3205.169
450	3281.074	460	3356.987
470	3433.305	480	3509.458
490	3585.864	500	3662.067
510	3738.388	520	3814.723
530	3891.239	540	3967.940
550	4044.805	560	4121.453
570	4198.256	580	4275.186
590	4352.266	600	4429.332
610	4506.490	620	4583.755
630	4660.923	640	4738.215
650	4815.714	660	4892.867
670	4970.318	680	5047.815
690	5125.640	700	5203.497

Here r_N and r_0 are the positions of the Nth monomer and the zeroth monomer respectively. In addition we have calculated the mean displacement $X_N - X_0$ in the X direction in order to check the quality of the data. Ideally one expects this quantity to be zero. Using this deviation we have typically found a discrepancy of smaller than 0.01% of $\langle R^2(N)\rangle^{1/2}$, showing the high accuracy of the data. The calculations have been performed in IBM extended precision (real *16).

3. Results

To calculate ν, the correlation length exponent, we use a standard method from series analysis (see, e.g., Djordjevic *et al* 1983). This method has been used by us (Kremer and Lyklema 1985b, Lyklema and Kremer 1985) successfully to analyse high accuracy

Monte Carlo data of different self-avoiding walk problems. The basic underlying assumption is about the asymptotic behaviour of the mean square end-to-end distance

$$\langle R^2(N)\rangle = AN^{2\nu}(1+BN^{-\Delta}+CN^{-1}+\ldots) \tag{4}$$

(see, e.g., Privman 1984). The conventional way of analysing Monte Carlo data is to ignore the correction to scaling and calculate ν from the slope of the plot of $\log\langle R^2(N)\rangle$ against $\log N$. Because a log-log plot tends to smooth the curvature it is very difficult to decide from such a plot if an asymptotic analysis is valid or not. A more sophisticated analysis can be made if we define an effective exponent $\nu(N)$

$$\nu(N) = \tfrac{1}{2}\frac{\log[\langle R^2(N+i)\rangle/\langle R^2(N-i)\rangle]}{\log[(N+i)/(N-i)]}. \tag{5}$$

If one inserts (4) in this definition the result is

$$\nu(N) = \nu - \tfrac{1}{2}BN^{-\Delta} - \tfrac{1}{2}CN^{-1}+\ldots. \tag{6}$$

From a plot of $\nu(N)$ against $N^{-\Delta}$ or N^{-1}, depending on whether Δ is smaller or larger than one, one can estimate the value of ν very accurately by a linear extrapolation of $\nu(N)$ for N values which are in the asymptotic scaling regime. To use this method for the analysis of Monte Carlo data instead of *exact* enumeration results, one needs very high accuracy data in order to minimise the statistical scatter in $\nu(N)$. This analysis can be improved significantly if one chooses a relative large i value in (5) (see, e.g., Lyklema and Kremer 1985). The absolute error of the denominator of course is the same for all i values. However the relative error in $\nu(N)$ will improve with a factor i. Of course one has to check if a bias is introduced for large i values but due to the symmetric definition (5) this seems to occur only for such large i values that the expansion of the logarithm is not valid anymore. Obviously one cannot obtain good enough accuracy for very long chains. This is because the variance $[(\langle R^4\rangle - \langle R^2\rangle^2)^{1/2}/\langle R^2\rangle^2]$ does not vanish for these walks and it becomes very difficult to cover phase space sufficiently. For the conventional way of analysing this is an even more serious problem because then one needs results for much larger N values. Also one has no criterion to decide if the asymptotic regime is reached already.

The exponent γ is calculated from the partition function which for a finite number of steps (N) is defined as

$$Z(N) = \sum_{C_N}\prod_{i=1}^{N} P(i, C_N). \tag{7}$$

$P(i, C_N)$ is the one-step probability for the ith step of a N-step walk with configuration C_N. For an exact enumeration one knows the exact probabilities and the exact number of configurations. This cannot be calculated in a Monte Carlo sampling. However it is obvious that the probability of having a chain of length N equals the acceptance rate $A(N)$

$$A(N) = \frac{\text{number of generated walks of length } N}{\text{number of attempts to generate a walk of length } N}. \tag{8}$$

Now we make the identification $Z(N) = A(N)$ and assume the same asymptotic behaviour as for the usual SAW (de Gennes 1979)

$$Z(N) \sim [(q_{\text{eff}})/q_0]^N N^{\gamma-1}. \tag{9}$$

The exponent γ is calculated from the slope of a log-log plot of $Z(N+i)/Z(N-i)$ against $(N+i)/(N-i)$.

$$\log \frac{Z(N+i)}{Z(N-i)} = 2i \log \frac{q_{\text{eff}}}{q_0} + (\gamma - 1) \log \frac{N+i}{N-i}$$

$$\underset{N\to\infty}{=} 2i \log \frac{q_{\text{eff}}}{q_0}. \tag{10}$$

By extrapolating this curve we also find immediately an estimate for the value of q_{eff}. To eliminate the effect of the constant in (10) one can study

$$\log \frac{Z(N+i)}{Z(N)} - \log \frac{Z(N)}{Z(N-1)} = (\gamma - 1) \log \frac{(N+i)(N-i)}{N^2}$$

$$\simeq (1-\gamma) i^2 / N^2. \tag{11}$$

As the extrapolation plotted against $1/N^2$ has to go through the origin, one can check in this way if the asymptotic analysis is valid.

3.1. 2D GSAW

To compare the two methods of analysing the mean square end-to-end distance we present in figure 2 the corresponding plots. The estimate for ν from the slope of figure 2(a) is 0.70. This value is obtained from chains with a maximum length of $N = 200$ on the square lattice. In an earlier Monte Carlo study Majid *et al* (1984a, b) found $\nu = 0.68$ from chains with a maximum length of $N = 350$ on the same lattice, a length for which the 'effective' exponent $\nu(N)$ is approximately 0.72. The difference is possibly due to the insufficient Monte Carlo quality of their data. As already discussed in the presentation of the method of analysis, one needs a very large number of samples in order to obtain a reliable result and certainly a few thousand chains are not sufficient for an accurate analysis. This can be understood from the asymptotic value ($N = 200$) of the variance. On the square lattice we find

$$\left(\frac{\langle R^4(N) \rangle - \langle R^2(N) \rangle^2}{\langle R^2(N) \rangle} \right)^{1/2} = 0.746.$$

This high value, which shows the non-vanishing variance and the width of the distribution function, illustrates again the need for high precision Monte Carlo data of relatively short chains, to make a reliable analysis possible. The other drawback of this method is also clear from figure 2(a); it is impossible to judge if the asymptotic regime has been reached. Only then is this analysis valid. In figure 2(b) we have shown the result of the alternative method. We have plotted $\nu(N)$ with $i = 5$ as defined in (5) for N values between 40 and 200. Compared with the same analysis with $i = 1$ this does not introduce a bias, but it increases the accuracy. The value of $\nu(N)$ is continuously increasing, starting at 0.65 for $N = 40$. For $N = 200$ one finds $\nu = 0.70$, in agreement with the result from figure 2(a). The advantage of the method is clear; one sees immediately that this value is not correct but is likely to extrapolate to $\nu = \frac{3}{4}$, because the slope is still increasing. To give a full numerical analysis, which would show that a *linear* extrapolation results in $\nu = 0.75$, one has to generate chains up to a length $N \sim 500$. We estimate that only for these long chains can the asymptotic behaviour

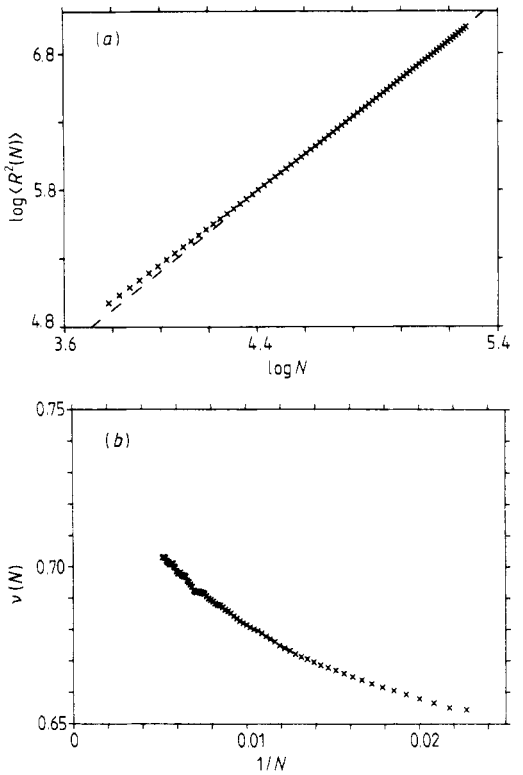

Figure 2. (*a*) A log-log plot of $\langle R^2(N)\rangle$ against N for N values between 40 and 200 on the square lattice. From the slope (broken line) one finds $\nu \simeq 0.70$. (*b*) A plot of $\nu(N)$ calculated from $\langle R^2(N)\rangle$ (equation (5)) with $i=5$ against $1/N$ for n values between 40 and 200 on the square lattice.

be studied and the correction to scaling exponent obtained. That one needs this correction to scaling exponent Δ to extrapolate correctly has been shown for the usual SAW (Djordjevic *et al* 1983, Lyklema and Kremer 1985). Using the same value for Δ (0.84) we found for the SAW did not result in a satisfactory extrapolation. This shows that the asymptotic scaling regime has not been reached yet. That an extrapolation of $\nu(N)$ in figure 2(*b*) cannot give a larger ν value than $\frac{3}{4}$ follows from the observation that always $\langle R^2(N)\rangle_{\mathrm{GSAW}} < \langle R^2(N)\rangle_{\mathrm{SAW}}$. Therefore the SAW value $\nu = \frac{3}{4}$ is an upper limit. A similar analysis for the fourth moment gives the same result.

The other quantity of interest is the exponent γ which governs the asymptotic behaviour of the partition function (see equation (9)). In figure 3 we show a log-log plot of $Z(N+1)/Z(N-1)$ against $(N+1)/(N-1)$. From the slope we estimate $\gamma = 1.34$. This value is confirmed by the analysis according to (11). However due to the additional difference in this expression, the relative error is much larger. This also explains why one cannot calculate the correction to scaling exponent from these data, although the asymptotic scaling regime apparently has been reached. If one adds a correction term $(1+BN^{-\Delta})$ to (9), this results in an additional term $-2i\Delta BN^{-(\Delta+1)}$ in (10). This we cannot observe because of the leading $(\gamma-1)/N$ behaviour. In equation (11) the $1/N$ term has disappeared, but now the fluctuations in the data are too large to distinguish between $1/N^2$ and $1/N^{\Delta+1}$ with $\Delta < 1$. Finally averaging the slopes

calculated from (10) for i values 10, 14 and 18 in the range $110 < N < 190$ results in an estimate of $\gamma = 1.343 \pm 0.003$. This is in excellent agreement with the SAW value $\frac{43}{32} \simeq 1.344$ (Nienhuis 1982). From an extrapolation of the data in figure 3 we find for the connectivity constant $q_{\text{eff}} = 2.934$ a value which has been found already from the exact enumeration results (Lyklema and Kremer 1984). This value, which is much larger than the SAW value 2.638 15 (Enting and Guttmann 1985), reflects the ability of the GSAW to avoid traps.

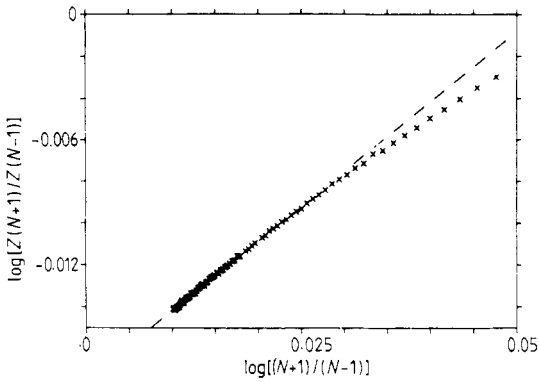

Figure 3. A log-log plot of $Z(N+1)/Z(N-1)$ against $(N+1)/(N-1)$ for N values between 40 and 200 on the square lattice. From the slope (broken line) one finds $\gamma = 1.34$.

3.2. 3D GSAW

In three dimensions the situation for a similar numerical analysis is not as favourable. Although the number of nearest neighbours on the diamond lattice is the same as on the square lattice, it is much more difficult to create a trap. On the square lattice the walk can get trapped already at the eighth step whereas on the dimaond lattice it takes 14 steps before trapping can occur. Following Pietronero (1984) we estimate the order of magnitude of the trapping probability for the square lattice to be $(\frac{1}{3})^4 = \frac{1}{81}$ and for the diamond lattice $(\frac{1}{3})^6 = \frac{1}{729}$. This prediction agrees very well with the numerical result in two dimensions of Hemmer and Hemmer (1984). Thus in order to see the asymptotic behaviour one needs chains much longer than 80 bonds on the square lattice and 700 bonds on the diamond lattice. However in the latter case one expects to see at least a bending towards the expected asymptotic result (0.59) in a plot of $\nu(N)$ against $1/N$ at $N \sim 700$. From Pietronero's argument it is also clear that only on the diamond lattice can one expect to see a glimpse of the asymptotic behaviour. From this discussion it is clear that similar convincing results as for the 2D walk cannot be given in three dimensions. In figure 4 we show a plot of $\nu(N)$ against $1/N$ for $180 < N < 700$. Again $\nu(N)$ is calculated using (5), now with $i = 13$. Comparison with an identical figure for $i = 1$ showed that this large i value does not introduce a bias but only increases the accuracy. From this figure we estimate that the error in $\nu(N)$ is ± 0.002. This is small enough to show that after $N > 400$ there is a clear trend upward to the expected value $\nu = 0.59$. For $N = 700$ we find $\nu(N) \sim 0.523$ up from a 'plateau' value somewhat smaller than 0.521. We estimate that in order to see a $\nu(N)$ value 0.53 one has to study chains of length $N \sim 2500$.

This is a length for which it is already hard to get good statistics. Thus it is impossible to study the real asymptotic behaviour of the GSAW in three dimensions

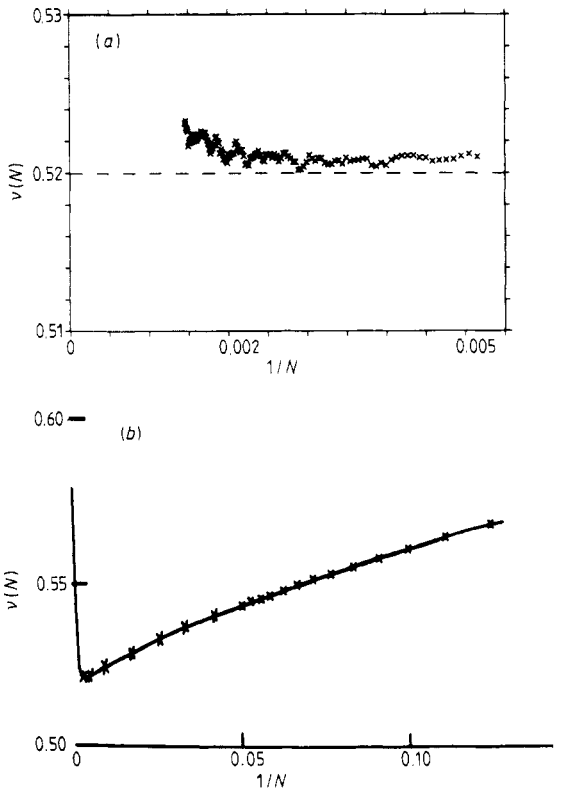

Figure 4. (*a*) A plot of $\nu(N)$ calculated from $\langle R^2(N)\rangle$ (equation (5)) with $i=13$ plotted against $1/N$ for N values between 180 and 700 on the diamond lattice. (*b*) $\nu(N)$ is shown for all N values. The crosses indicate measured points. For $N>700$ the steep increase shows the *expected* behaviour ($\nu \simeq 0.59$).

numerically. Also the study of the partition function $Z(N)$ (equation (9)) for $N<700$ does not give information about the asymptotic behaviour. However the slope $(\gamma-1)$ of equation (10) has increased to ~ 0.04 for $N\sim 700$, excluding $\gamma=1$. These results clearly exclude the possibility of an upper critical dimension $d_c=3$.

4. Discussion

We have presented a detailed numerical study of the recently introduced GSAW. We have used a ratio method to analyse high accuracy Monte Carlo data, extending the exact enumeration results for $N\leqslant 22$ to N values of 200 (2D) and 700 (3D) respectively. This method, which has been used successfully for the indefinitely growing SAW (Kremer and Lyklema 1985a, b) and the usual SAW (Lyklema and Kremer 1985), gives also here substantially more information about the system. As opposed to a simple log-log plot (see figure 2(*a*)) which gives an estimate for ν alone, we obtain a sequence of values for $\nu(N)$ (equation (5)). From this it is usually possible to give a very accurate estimate for ν provided one has data which are accurate enough to apply this analysis. Here, as in the conventional log-log plot analysis one needs chains long enough to cover

the asymptotic regime. But even if this is not the case, and one obtains a wrong estimate from a log-log plot analysis, one can make a rough estimate for ν from a ratio type analysis because the trend is known. For the GSAW on the square lattice this would lead to an estimate of $\nu = 0.75 \pm 0.01$. This is in strong contradiction with the earlier results from exact enumerations (Lyklema and Kremer 1984b) and Monte Carlo simulations (Majid *et al* 1984a, b). It also shows clearly that the series analysis does not get into the asymptotic regime and therefore cannot give a correct result. This result has already been anticipated by Peliti (1984) and Pietronero (1984). Both authors argue that this walk has still a small but non-zero probability to get trapped. From this they conclude, using field theory and a self-consistent mean field type theory respectively, that the GSAW is in the SAW universality class. This prediction is nicely confirmed by our estimate for the exponent ν whereas the result for the exponent γ is in complete agreement with the SAW value $\frac{43}{32}$ (Nienhuis 1982). In three dimensions we have not been able to give similar good results. The sequence $\nu(N)$ however shows a minimum at $N \sim 400$ and then starts to increase, a behaviour which certainly excludes the possibility $\nu = \frac{1}{2}$ as suggested by Majid *et al* (1984). Also the still increasing value for the slope, which asymptotically would give $(\gamma - 1)$, does not support the notion of a critical dimension $d_c = 3$.

At this point it is appropriate to discuss a different model which has been introduced recently by Havlin *et al* (1984). The model, which is proposed to describe cluster growth for branched polymers, is parametrised by a branching parameter B. As noted by Havlin *et al* the choice $B = 1$ reduces this model to a model which is in the same universality class as the GSAW. In agreement with the findings of Majid *et al* (1984) they find $\nu = 0.68$. Remarkably enough they find this result for *all* values of the branching parameter B. From this they conclude that all these models belong to the same universality class. However, as is now known, this result for the GSAW is wrong and consequently the result 0.68 for the cluster-growth model with $B = 1$ should be wrong. This obviously raises questions about the validity of the results for other B values also. In our opinion it is quite possible that also for this model much larger clusters have to be studied in order to see the asymptotic behaviour. In addition, an analysis similar to the one presented in this paper seems to be necessary in order to decide if the observation that ν is independent of the branching parameter B is correct.

To summarise we have shown that the two-dimensional GSAW has the same asymptotic behaviour as the usual SAW. In three dimensions the asymptotic behaviour only shows up for very large N values and a complete numerical analysis is not possible. However it is shown that the exponent ν in three dimensions is *not* equal to $\frac{1}{2}$ and that therefore the upper critical dimension is not equal to three. From the approximate analytical theories (Peliti 1984, Pietronero 1984) one expects to find the SAW values for the exponents as in the two-dimensional case. The difference between the SAW and the GSAW only shows up in the connectivity constant q_{eff}, 2.64 against 2.94. Because the GSAW has on average more sites to jump to, it is more difficult to create trapping situations. This explains why the asymptotic behaviour for the GSAW shows up only for much larger N values.

References

Amit D J, Parisis G and Peliti L 1983 *Phys. Rev.* B **27** 1635

de Gennes P G 1979 *Scaling Concepts in Polymer Physics* (Ithaca, NY: Cornell University Press)

Djordjevic Z V, Majid I, Stanley H E and dos Santos R J 1983 *J. Phys. A: Math. Gen.* **16** L519
Enting J G and Guttmann A J 1985 *J. Phys. A: Math. Gen.* **18** 1007
Family F and Landau D P (ed) 1984 *Kinetics of Aggregation and Gelation* (Amsterdam: North-Holland)
Havlin S, Trus B and Stanley H E 1984 *Phys. Rev. Lett.* **53** 1288
Hemmer S and Hemmer P C 1984 *J. Chem. Phys.* **81** 584
Kremer K, Baumgärtner A and Binder K 1982 *J. Phys. A: Math. Gen.* **15** 2879
Kremer K and Lyklema J W 1985a *Phys. Rev. Lett.* **54** 267
—— 1985b *J. Phys. A: Math. Gen.* **18** 1515
Lyklema J W and Kremer K 1984a in *Kinetics of Aggregation and Gelation* ed F Family and D P Landau (Amsterdam: North-Holland)
—— 1984b *J. Phys. A: Math. Gen.* 7 L691
—— 1985 *Phys. Rev.* B **31** 3182
Majid I, Jan N, Coniglio A and Stanley H E 1984a in *Kinetics of Aggregation and Gelation* ed F Family and D P Landau (Amsterdam: North-Holland)
—— 1984b *Phys. Rev. Lett.* **52** 1257
Nienhuis B 1982 *Phys. Rev. Lett.* **49** 1062
Peliti L 1984 *J. Physique Lett.* **45** L925
Pietronero L 1984 *Preprint*
Privman V 1984 *Physica* **123A** 428